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The various versions of Bell's inequality: an alternative proof

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Abstract
By considering uncommon factors as spacetime events that influence the spin orientations in the EPRB thought experiment, it is intended to show that one can still introduce the correlation functions. These uncommon factors are positioned inside the common lightcone of two particles. Then, Bell inequalities are proved with the preassumptions of local realism and spin conservation law in the context of a new scenario of hidden variables.

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1. Introduction

Based on the well-known thought experiment of EPRB, Bell proved his famous inequality in 1964 by introducing some new ideas [1]. This inequality was derived by considering the locality and realism presuppositions. The realism assumption based on the well-known EPR paper has been admitted [2]. Einstein, Podolski and Rosen have shown that there are elements of physical reality that cannot be described by quantum mechanics. Bell in his explanation [1, 3] introduced an element of reality which cannot be represented by quantum mechanics, and represented it by λ as a hidden variable that determines spin orientation of the particle in the thought experiment of EPRB inspired by the idea of EPR. λ can contain one or more than one variable. Both these assumptions, i.e. locality and realism, play a central role in deriving Bell inequalities. Later, some mathematical inequalities were constructed by which empirical tests for distinguishing the two theories, quantum mechanics and local realistic theories, were innovated [4, 5].

In Bell's approach for the EPRB thought experiment, it is assumed that each particle with total spin zero decays into spin $1/2$ particles running away from each other along a straight line. Taking the spin conservation law into consideration, if we detect the spin of particle 1 (particle on the right) along some specific straight direction by means of detector D_A and the result, in terms of $\frac{\hbar}{2}$, turns out to be +1 (−1), then we conclude that for the spin of particle 2 (particle on the left) along the same direction by means of detector D_B the value −1 (+1) is measured. If we suppose that detectors D_A and D_B have been adjusted along $\hat{a}$ and $\hat{b}$ respectively, then we can consider the result of measurements on the right- and left-hand sides as

$$A = A(\hat{a}, \lambda), \qquad B = B(\hat{b}, \lambda). \tag{1}$$

The choice of different orientations $\hat{a}$ and $\hat{b}$ on both sides was also Bell's innovation. The allowed values for A and B in terms of $\frac{\hbar}{2}$ are ± 1. The realism assumption is applied through λ and is denoted as a label of a pair of particles in that upon changing λ, another pair of particles is considered. Although on the basis of the idea of hidden variables, the explicit functionality of particle spin, that is, A and B, of λ is unknown, it is necessary to let λ belong to some region Λ, such that the integral of a weight function such as $\rho(\lambda) \geqslant 0$ is normalized to 1:

$$\int_\Lambda \rho(\lambda)\, \mathrm{d}\lambda = 1. \tag{2}$$

Despite λ being unknown, the fulfilment of condition (2) for summability (integrability) of the effects of λ is necessary. Besides realism, the assumption of locality has also been admitted in equations (1), since it has been assumed that A (B) is not influenced by the orientation of detector D_B (D_A), i.e. $\hat{b}$ ($\hat{a}$). Had the locality condition not held then we would have accepted that

$$A(\hat{a}, \lambda) = A(\hat{a}, \hat{b}, \lambda), \qquad B(\hat{b}, \lambda) = B(\hat{b}, \hat{a}, \lambda). \tag{3}$$

The existence of hidden variables λ has the outcome that they generate a correlation between the spin values of particles on the right and left:

$$C(\hat{a}, \hat{b}) = \int_\Lambda A(\hat{a}, \lambda) B(\hat{b}, \lambda) \rho(\lambda)\, \mathrm{d}\lambda. \tag{4}$$

By relying on the concept of realism as well as locality, Bell inequalities for correlation functions with different versions containing three and four directions have been derived as [6]

$$|C(\hat{a}, \hat{b}) - C(\hat{a}, \hat{b}')| \leqslant 1 + C(\hat{b}, \hat{b}') \tag{5a}$$

$$|C(\hat{a}, \hat{b}) - C(\hat{a}, \hat{b}') + C(\hat{a}', \hat{b}) + C(\hat{a}', \hat{b}')| \leqslant 2 \tag{5b}$$

$$|C(\hat{a}, \hat{b}) - C(\hat{a}, \hat{b}')| + |C(\hat{a}', \hat{b}) + C(\hat{a}', \hat{b}')| \leqslant 2. \tag{5c}$$

In the next step Bell showed that it is possible for these inequalities to contradict the predictions of quantum mechanics. This is a serious theoretical conflict between quantum mechanics and Bell inequality assumptions. But the situation became even more serious when Aspect *et al* (in 1982) performed experiments which, based upon quantum mechanics predictions, contradicted Bell inequalities [7, 8]. The contradiction of Bell inequalities puts a question mark on the assumption of inequalities, and this can have big outcomes such as incompatibility between the theory of quantum mechanics and the theory of special relativity. Of course one can also interpret this incompatibility between the theory of quantum mechanics and the theory of local hidden variables. In fact, many experiments have indicated that Bell inequalities are incompatible with quantum mechanics and compatible with local realism [7–20]. In the next section, we shall present a different and a new role played by the hidden variables, which has not escaped the eyes of John Bell either [6], such that on its basis one can prove the various versions of Bell's inequality in the presence of uncommon influencing causes.

2. A new scenario for hidden variables and correlation functions

In the approach mentioned in section 1, all common influencing causes in the common lightcone on the destiny of a particle pair are shown with a latent label λ, and a change in λ means a change in the particle pair. In fact, λ acts as an ID card for a particle pair. In this section, we look at the realism considered by λ differently, that is, we suppose that the spins of the right and the left particles reached at the detectors D_A and D_B are influenced by a set of common causes in the common lightcone, labelled by Λ. The region Λ contains some hidden variables λ in the common lightcone which are considered as the causal events influencing the spins of both particles on the right and on the left. In fact, this time Λ acts as an ID card for the particle pair. $\rho(\lambda) \geqslant 0$ is the weight function corresponding to the influence of all common hidden variables λ belonging to Λ on the spin of pair particles, normalized to 1 as in equation (2). We can also pose this question as to whether there are any uncommon causes such as η and ζ, as some elements of reality, which have roles in the determination of A and B. The answer to this question is not necessarily negative. Therefore, one does not have a strong reason to neglect them. The uncommon causal events η and ζ are located inside the common lightcone and influence locally the spin values of A and B, respectively. In the framework of this proposal, the spin values of particles on the right and left are determined by η and λ, as well as ζ and λ, respectively [10, 21]:

$$A = A(\hat{a}, \eta, \lambda), \qquad B = B(\hat{b}, \zeta, \lambda). \tag{6}$$

In equations (6) η and ζ are the uncommon causal events effective in determining A and B respectively, which themselves can contain more than one continuous or discrete variable (event). η and ζ may not exist at all or they may be the events $\hat{b}$ and $\hat{a}$ respectively, that is, the measuring angles on opposite sides. As in the case of λ where we were not able to understand the details of its influence on the values of spin of particles and we accepted its mere existence, in the case of η and ζ too they are such uncommon probabilistic influencing agents. In order to indicate the correlation between the spin values along the right and the left angular directions based on the description given in the previous section of the variable λ, we are obliged to speak of more than one pair of particles. Whereas in the description of the present section, in terms of λ, the correlation between spin values along the right and the left directions also comes into existence in an experiment with a pair of particles. It is exactly due to this new scheme of hidden variables that we can bring in the uncommon factors η and ζ, influencing the spin of particles 1 and 2 respectively.

Henceforth, we consider the corresponding correlation functions as a product of spins of particles 1 and 2 in different set-ups for the right and the left detectors with angles $\hat{a}$ and $\hat{b}$, $\hat{a}$ and $\hat{b}'$, $\hat{b}$ and $\hat{b}'$, $\hat{a}'$ and $\hat{b}$, and finally $\hat{a}'$ and $\hat{b}'$ in the following way:

$$C_{\eta\zeta}(\hat{a}, \hat{b}) = \int_\Lambda A(\hat{a}, \eta, \lambda) B(\hat{b}, \zeta, \lambda) \rho(\lambda)\, \mathrm{d}\lambda \tag{7a}$$

$$C_{\eta'\zeta'}(\hat{a}, \hat{b}') = \int_\Lambda A(\hat{a}, \eta', \lambda) B(\hat{b}', \zeta', \lambda) \rho(\lambda)\, \mathrm{d}\lambda \tag{7b}$$

$$C_{\eta''\zeta''}(\hat{b}, \hat{b}') = \int_\Lambda A(\hat{b}, \eta'', \lambda) B(\hat{b}', \zeta'', \lambda) \rho(\lambda)\, \mathrm{d}\lambda \tag{7c}$$

$$C_{\eta''\zeta''}(\hat{a}', \hat{b}) = \int_\Lambda A(\hat{a}', \eta'', \lambda) B(\hat{b}, \zeta'', \lambda) \rho(\lambda)\, \mathrm{d}\lambda \tag{7d}$$

$$C_{\eta'''\zeta'''}(\hat{a}', \hat{b}') = \int_\Lambda A(\hat{a}', \eta''', \lambda) B(\hat{b}', \zeta''', \lambda) \rho(\lambda)\, \mathrm{d}\lambda. \tag{7e}$$

Relations (7a)–(7c) and (7a), (7b), (7d), (7e) will be used in obtaining the Bell inequalities containing spin measurements in three and four directions, respectively. By choosing different configurations for detecting set-ups, it is clear that for a pair of particles 1 and 2, the influencing hidden common factors, Λ, must not change. But the use of different subscripts in each of equations (7a)–(7e) means that the change of uncommon factors from one thought experiment to another has been allowed. If we suppose that $\eta = \eta' = \eta'' = \eta'''$ and $\zeta = \zeta' = \zeta'' = \zeta'''$, one may expect that in all five thought experiments the uncommon factors are the same, in which case writing them seems to be unnecessary. Since the values of A and B in terms of $\frac{\hbar}{2}$ are ± 1, and they describe the spin of particles 1 and 2 in different directions, we expect them to have the following mathematical properties:

$$|A(\hat{a},\eta,\lambda)| = |A(\hat{a},\eta',\lambda)| = |A(\hat{b},\eta'',\lambda)| = |B(\hat{b},\zeta,\lambda)| = |B(\hat{b}',\zeta',\lambda)| = \cdots = 1 \tag{8}$$

$$A(\hat{a},\eta,\lambda)B(\hat{a},\zeta,\lambda) = A(\hat{b},\eta,\lambda)B(\hat{b},\zeta,\lambda) = A(\hat{a}',\eta',\lambda)B(\hat{a}',\zeta',\lambda) = \cdots = -1. \tag{9}$$

Equations (9) describe the spin conservation law. Although these equations do not show explicitly the functionality of spin from factors $\hat{a}, \hat{b}, \eta, \zeta, \lambda, \ldots$ as such, they give the useful results obtained in the next section. It must be recalled that the existence of local hidden variables models is a sufficient but not necessary condition for deriving the Bell inequalities (for instance, see [22]).

3. Spin conservation law in conjunction with local realism towards Bell inequalities

Now we are in a position that for the correlation functions given in equations (7) we can derive three- and four-angle Bell inequality versions. Although the functionality of spin values of particles 1 and 2, that is A and B, of their arguments is unclear to us, we can obtain general results from the spin conservation law, i.e. (9), which leads to Bell inequalities. To this end note that from equations (9) we can get

$$\frac{A(\hat{a},\eta,\lambda)}{A(\hat{b},\eta,\lambda)} = \frac{B(\hat{b},\zeta,\lambda)}{B(\hat{a},\zeta,\lambda)} =: u(\hat{a},\hat{b},\lambda). \tag{10}$$

Due to the expression on the right-hand side, equation (10) must be independent of η, and due to the expression on the left-hand side, equation (10) must be independent of ζ. Thus we indicate it by $u(\hat{a},\hat{b},\lambda)$. By considering equations (8) and (10) one obtains the following three important and determining properties for the functions $u(\hat{a},\hat{b},\lambda)$:

$$u(\hat{a},\hat{a},\lambda) = 1 \tag{11a}$$

$$u(\hat{a},\hat{b},\lambda) = u(\hat{b},\hat{a},\lambda) \tag{11b}$$

$$u(\hat{a},\hat{b},\lambda)u(\hat{b},\hat{b}',\lambda) = u(\hat{a},\hat{b}',\lambda). \tag{11c}$$

It is clear that the allowed values for $u(\hat{a},\hat{b},\lambda)$ are ± 1. Equations (11a)–(11c) for measuring directions of spin values give an equivalence relation through u. They represent the reflexive, symmetric and transitive properties of the equivalence relation satisfied by the function u, respectively. Hence, the different measuring spin directions are put in the same equivalence class. This is a direct consequence of the spin conservation law. Note that one can also obtain this result without considering uncommon causes η and ζ etc. In the next stage, we show by two different methods that the mere anticipation of the stated fact, equations (11), leads to the Bell inequalities.

In order to prove Bell inequalities with three different directions for measuring the spin of particles 1 and 2, let us apply equations (7*a*) and (7*b*) and write

$$C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}') = \int_\Lambda A(\hat{a},\eta,\lambda)B(\hat{b},\zeta,\lambda)\left[1 - \frac{A(\hat{a},\eta',\lambda)B(\hat{b}',\zeta',\lambda)}{A(\hat{a},\eta,\lambda)B(\hat{b},\zeta,\lambda)}\right]\rho(\lambda)\,\mathrm{d}\lambda. \tag{12}$$

With $\rho(\lambda) \geqslant 0$ and by considering equations (8) one can find the following result for the modules of equation (12):

$$|C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}')| \leqslant \int_\Lambda \left|1 - \frac{A(\hat{a},\eta',\lambda)B(\hat{b}',\zeta',\lambda)}{A(\hat{a},\eta,\lambda)B(\hat{b},\zeta,\lambda)}\right|\rho(\lambda)\,\mathrm{d}\lambda. \tag{13}$$

With the repetitive application of equations (9) and (10), we can obtain

$$\begin{aligned}
|C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}')| &\leqslant \int_\Lambda \left|1 - \frac{u(\hat{a},\hat{b},\lambda)A(\hat{b},\eta',\lambda)B(\hat{b}',\zeta',\lambda)}{u(\hat{a},\hat{b},\lambda)A(\hat{b},\eta,\lambda)B(\hat{b},\zeta,\lambda)}\right|\rho(\lambda)\,\mathrm{d}\lambda \\
&= \int_\Lambda \left|1 - \frac{A(\hat{b},\eta',\lambda)B(\hat{b}',\zeta',\lambda)}{A(\hat{b}',\eta',\lambda)B(\hat{b}',\zeta',\lambda)}\right|\rho(\lambda)\,\mathrm{d}\lambda \\
&= \int_\Lambda \left|1 - \frac{A(\hat{b},\eta',\lambda)B(\hat{b}',\zeta'',\lambda)}{A(\hat{b}',\eta',\lambda)B(\hat{b}',\zeta'',\lambda)}\right|\rho(\lambda)\,\mathrm{d}\lambda \\
&= \int_\Lambda \left|1 - \frac{u(\hat{b},\hat{b}',\lambda)A(\hat{b}',\eta',\lambda)B(\hat{b}',\zeta'',\lambda)}{A(\hat{b}',\eta',\lambda)B(\hat{b}',\zeta'',\lambda)}\right|\rho(\lambda)\,\mathrm{d}\lambda \\
&= \int_\Lambda \left|1 - \frac{u(\hat{b},\hat{b}',\lambda)A(\hat{b}',\eta'',\lambda)B(\hat{b}',\zeta'',\lambda)}{A(\hat{b}',\eta'',\lambda)B(\hat{b}',\zeta'',\lambda)}\right|\rho(\lambda)\,\mathrm{d}\lambda \\
&= \int_\Lambda |1 + A(\hat{b},\eta'',\lambda)B(\hat{b}',\zeta'',\lambda)|\rho(\lambda)\,\mathrm{d}\lambda.
\end{aligned} \tag{14}$$

Consider the inequality $A(\hat{b},\eta'',\lambda)B(\hat{b}',\zeta'',\lambda) \geqslant -1$ and use equations (2) and (7*c*). Then, from (14) we obtain

$$|C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}')| \leqslant 1 + C_{\eta''\zeta''}(\hat{b},\hat{b}'). \tag{15}$$

Relation (15) is a description of Bell's inequality with three different angles for correlation functions corresponding to a pair of particles. These correlation functions are related to three different set-ups $\hat{a}$ and $\hat{b}$, $\hat{a}$ and $\hat{b}'$, $\hat{b}$ and $\hat{b}'$ which here become influenced, respectively, by the independent events η and ζ, η' and ζ', η'' and ζ''.

Now we can also obtain the inequalities corresponding to the correlation functions with four different directions for measuring spin values. To this end, note that we have

$$\begin{aligned}
C_{\eta\zeta}(\hat{a},\hat{b}) - C_{\eta'\zeta'}(\hat{a},\hat{b}') &= \int_\Lambda (A(\hat{a},\eta,\lambda)B(\hat{b},\zeta,\lambda) - A(\hat{a},\eta',\lambda)B(\hat{b}',\zeta',\lambda))\rho(\lambda)\,\mathrm{d}\lambda \\
&\quad \pm \int_\Lambda u(\hat{a},\hat{b},\lambda)A(\hat{b},\eta,\lambda)B(\hat{b},\zeta,\lambda)u(\hat{a}',\hat{b}',\lambda)A(\hat{b}',\eta''',\lambda)B(\hat{b}',\zeta''',\lambda)\rho(\lambda)\,\mathrm{d}\lambda \\
&\quad \mp \int_\Lambda u(\hat{a},\hat{b},\lambda)A(\hat{b},\eta',\lambda)B(\hat{b},\zeta'',\lambda)u(\hat{a}',\hat{b}',\lambda)A(\hat{a}',\eta'',\lambda)B(\hat{a}',\zeta',\lambda)\rho(\lambda)\,\mathrm{d}\lambda \\
&= \int_\Lambda A(\hat{a},\eta,\lambda)B(\hat{b},\zeta,\lambda)(1 \pm A(\hat{a}',\eta''',\lambda)B(\hat{b}',\zeta''',\lambda))\rho(\lambda)\,\mathrm{d}\lambda \\
&\quad - \int_\Lambda A(\hat{a},\eta',\lambda)B(\hat{b}',\zeta',\lambda)(1 \pm A(\hat{a}',\eta'',\lambda)B(\hat{b},\zeta'',\lambda))\rho(\lambda)\,\mathrm{d}\lambda.
\end{aligned} \tag{16}$$

Table 1. Different possible ways of allocating the allowed values +1 and −1 to the function u, while the spins are measured in three different directions $\hat{a}$, $\hat{b}$ and $\hat{b}'$. There are four such ways.

$u(\hat{a}, \hat{b}, \lambda)$	$u(\hat{a}, \hat{b}', \lambda)$	$u(\hat{b}, \hat{b}', \lambda)$
+1	+1	+1
+1	−1	−1
−1	+1	−1
−1	−1	+1

As before, by calculating the modules of (16) one can easily obtain

$$|C_{\eta\zeta}(\hat{a}, \hat{b}) - C_{\eta'\zeta'}(\hat{a}, \hat{b}')| \leqslant 2 \pm (C_{\eta''\zeta''}(\hat{a}', \hat{b}) + C_{\eta'''\zeta'''}(\hat{a}', \hat{b}')). \tag{17}$$

One can easily derive from inequalities (17) the following two different forms for the Bell inequalities with four different spin measuring directions:

$$|C_{\eta\zeta}(\hat{a}, \hat{b}) - C_{\eta'\zeta'}(\hat{a}, \hat{b}') + C_{\eta''\zeta''}(\hat{a}', \hat{b}) + C_{\eta'''\zeta'''}(\hat{a}', \hat{b}')| \leqslant 2 \tag{18a}$$

$$|C_{\eta\zeta}(\hat{a}, \hat{b}) - C_{\eta'\zeta'}(\hat{a}, \hat{b}')| + |C_{\eta''\zeta''}(\hat{a}', \hat{b}) + C_{\eta'''\zeta'''}(\hat{a}', \hat{b}')| \leqslant 2. \tag{18b}$$

Arbitrariness of the subscripts in relations (15) and (18) is considered as an excellent symmetry for the Bell inequalities. One can, out of ignorance, neglect the subscripts, but note that not writing them does not imply their non-existence.

Note that in the first method, as described above, we have made use of the properties (11) analytically. In the second method, which we are about to explain, we will make use of (11) differently. To derive the Bell inequalities with three different measuring directions $\hat{a}$, $\hat{b}$ and $\hat{b}'$ we will deal with the functions $u(\hat{a}, \hat{b}, \lambda)$, $u(\hat{a}, \hat{b}', \lambda)$ and $u(\hat{b}, \hat{b}', \lambda)$. A simple inspection shows that one can allocate in four different ways the values +1 and −1 to these three functions such that at the same time equations (11c) are also satisfied. These four states are illustrated in table 1. Table 1 indicates that one obtains the following result for all four states:

$$|-u(\hat{a}, \hat{b}, \lambda) + u(\hat{a}, \hat{b}', \lambda)| + u(\hat{b}, \hat{b}', \lambda) = 1. \tag{19}$$

Now, from equation (19) and equations (7a)–(7c) one simply obtains

$$|C_{\eta\zeta}(\hat{a}, \hat{b}) - C_{\eta'\zeta'}(\hat{a}, \hat{b}')| - C_{\eta''\zeta''}(\hat{b}, \hat{b}') \leqslant \int_\Lambda (|-u(\hat{a}, \hat{b}, \lambda) + u(\hat{a}, \hat{b}', \lambda)| + u(\hat{b}, \hat{b}', \lambda))\rho(\lambda)\mathrm{d}\lambda = 1. \tag{20}$$

This is the Bell inequality (15) that corresponds to the measurement of spin values in three different directions. The Bell inequalities in which spin values in four different directions $\hat{a}, \hat{a}', \hat{b}$ and $\hat{b}'$ are measured can be obtained accordingly. A simple inspection shows that one can allocate in eight different ways the values +1 and −1 to the six functions $u(\hat{a}, \hat{a}', \lambda), u(\hat{b}, \hat{b}', \lambda), u(\hat{a}, \hat{b}, \lambda), u(\hat{a}, \hat{b}', \lambda), u(\hat{a}', \hat{b}, \lambda)$ and $u(\hat{a}', \hat{b}', \lambda)$ such that at the same time equations (11c) are also satisfied. These eight states are illustrated in table 2. From table 2 one can conclude that the following quantities have the constant value 2 in those eight states:

$$|-u(\hat{a}, \hat{b}, \lambda) + u(\hat{a}, \hat{b}', \lambda) - u(\hat{a}', \hat{b}, \lambda) - u(\hat{a}', \hat{b}', \lambda)| = 2 \tag{21a}$$

$$|u(\hat{a}, \hat{b}, \lambda) - u(\hat{a}, \hat{b}', \lambda)| + |u(\hat{a}', \hat{b}, \lambda) + u(\hat{a}', \hat{b}', \lambda)| = 2. \tag{21b}$$

Now taking into consideration equations (10) and (9) we have

$$C_{\eta\zeta}(\hat{a}, \hat{b}) - C_{\eta'\zeta'}(\hat{a}, \hat{b}') + C_{\eta''\zeta''}(\hat{a}', \hat{b}) + C_{\eta'''\zeta'''}(\hat{a}', \hat{b}') = \int_\Lambda (-u(\hat{a}, \hat{b}, \lambda) + u(\hat{a}, \hat{b}', \lambda) - u(\hat{a}', \hat{b}, \lambda) - u(\hat{a}', \hat{b}', \lambda))\rho(\lambda)\,\mathrm{d}\lambda. \tag{22}$$

Table 2. Different possible ways allocating the allowed values +1 and −1 to the function u, while the spins are measured in four different directions $\hat{a}$, $\hat{b}$, $\hat{a}'$ and $\hat{b}'$. There are eight such ways.

$u(\hat{a}, \hat{a}', \lambda)$	$u(\hat{b}, \hat{b}', \lambda)$	$u(\hat{a}, \hat{b}, \lambda)$	$u(\hat{a}, \hat{b}', \lambda)$	$u(\hat{a}', \hat{b}, \lambda)$	$u(\hat{a}', \hat{b}', \lambda)$
+1	−1	−1	+1	−1	+1
+1	−1	+1	−1	+1	−1
+1	+1	−1	−1	−1	−1
+1	+1	+1	+1	+1	+1
−1	−1	−1	+1	+1	−1
−1	−1	+1	−1	−1	+1
−1	+1	−1	−1	+1	+1
−1	+1	+1	+1	−1	−1

If we calculate the modules of both sides of (22) and then make use of (21a) we get inequality (18a) at once. Also, with the aid of equations (10) and (9) we have

$$C_{\eta\zeta}(\hat{a}, \hat{b}) - C_{\eta'\zeta'}(\hat{a}, \hat{b}') = \int_\Lambda (-u(\hat{a}, \hat{b}, \lambda) + u(\hat{a}, \hat{b}', \lambda))\rho(\lambda)\, \mathrm{d}\lambda \tag{23a}$$

$$C_{\eta''\zeta''}(\hat{a}', \hat{b}) + C_{\eta'''\zeta'''}(\hat{a}', \hat{b}') = \int_\Lambda (-u(\hat{a}', \hat{b}, \lambda) - u(\hat{a}', \hat{b}', \lambda))\rho(\lambda)\, \mathrm{d}\lambda. \tag{23b}$$

If we calculate the modules of both sides of (23a) and (23b), then sum them up and finally make use of (21b), we obtain inequality (18b). We note that the functions u together with their specifications, (8) and (9), lead to an alterative derivation of Bell inequalities whatever method one uses.

Now we show that Bell inequalities can also be derived for N source particles (or N particle pairs). To this end, we label every pair of particles under experiment by i through their effective uncommon events and define the correlation functions for N particle pairs as the mean of correlation functions of all particle pairs:

$$\begin{aligned}
C(\hat{a}, \hat{b}) &:= \frac{1}{N}\sum_{i=1}^{N} C_{\eta_i\zeta_i}(\hat{a}, \hat{b}) \\
C(\hat{a}, \hat{b}') &:= \frac{1}{N}\sum_{i=1}^{N} C_{\eta_i'\zeta_i'}(\hat{a}, \hat{b}') \\
C(\hat{a}', \hat{b}) &:= \frac{1}{N}\sum_{i=1}^{N} C_{\eta_i''\zeta_i''}(\hat{a}', \hat{b}) \\
C(\hat{a}', \hat{b}') &:= \frac{1}{N}\sum_{i=1}^{N} C_{\eta_i'''\zeta_i'''}(\hat{a}', \hat{b}').
\end{aligned} \tag{24}$$

These definitions result at once

$$\begin{aligned}
&|C(\hat{a}, \hat{b}) - C(\hat{a}, \hat{b}')| + |C(\hat{a}', \hat{b}) + C(\hat{a}', \hat{b}')| \\
&\quad = \frac{1}{N}\Big|\sum_{i=1}^{N}\big(C_{\eta_i\zeta_i}(\hat{a}, \hat{b}) - C_{\eta_i'\zeta_i'}(\hat{a}, \hat{b}')\big)\Big| + \frac{1}{N}\Big|\sum_{i=1}^{N}\big(C_{\eta_i''\zeta_i''}(\hat{a}', \hat{b}) + C_{\eta_i'''\zeta_i'''}(\hat{a}', \hat{b}')\big)\Big| \\
&\quad \leqslant \frac{1}{N}\sum_{i=1}^{N}\big(\big|C_{\eta_i\zeta_i}(\hat{a}, \hat{b}) - C_{\eta_i'\zeta_i'}(\hat{a}, \hat{b}')\big| + \big|C_{\eta_i''\zeta_i''}(\hat{a}', \hat{b}) + C_{\eta_i'''\zeta_i'''}(\hat{a}', \hat{b}')\big|\big).
\end{aligned} \tag{25}$$

By considering the fact that the Bell inequality (18b) for any of the ith particle pairs with correlation functions $C_{\eta_i\zeta_i}(\hat{a},\hat{b})$, $C_{\eta_i'\zeta_i'}(\hat{a},\hat{b}')$, $C_{\eta_i''\zeta_i''}(\hat{a}',\hat{b})$ and $C_{\eta_i'''\zeta_i'''}(\hat{a}',\hat{b}')$ holds, we conclude from relation (25) that

$$|C(\hat{a},\hat{b}) - C(\hat{a},\hat{b}')| + |C(\hat{a}',\hat{b}) + C(\hat{a}',\hat{b}')| \leqslant 2. \tag{26}$$

Likewise, one can obtain similar three- and four-angle inequalities (15) and (18a) for N particle pairs. Although relations (5a)–(5c) and (15), (18a), (18b) look the same, we must, however, note that the definitions of correlation functions in these relations are different.

Thus, according to the interpretation of the hidden variables presented in this paper, one can also derive the Bell inequalities for N particle pairs. Of course, the number of particle pairs need not be greater than 1, or even innumerable. One can claim that increasing the number of particle pairs serves the testability of Bell inequalities and will be used just for distributing the measurement errors. In the arrangement where the derivation of Bell inequalities for more than one particle was explained, this point is clear that the presence of local external influencing factors, e.g. η and ζ, is neither understandable nor testable. In other words, Bell inequalities are natural results of simultaneous acceptance of the idea of local realism Λ and spin conservation law. Thus, Bell inequalities are indifferent to the acceptance of the influence of such factors.

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